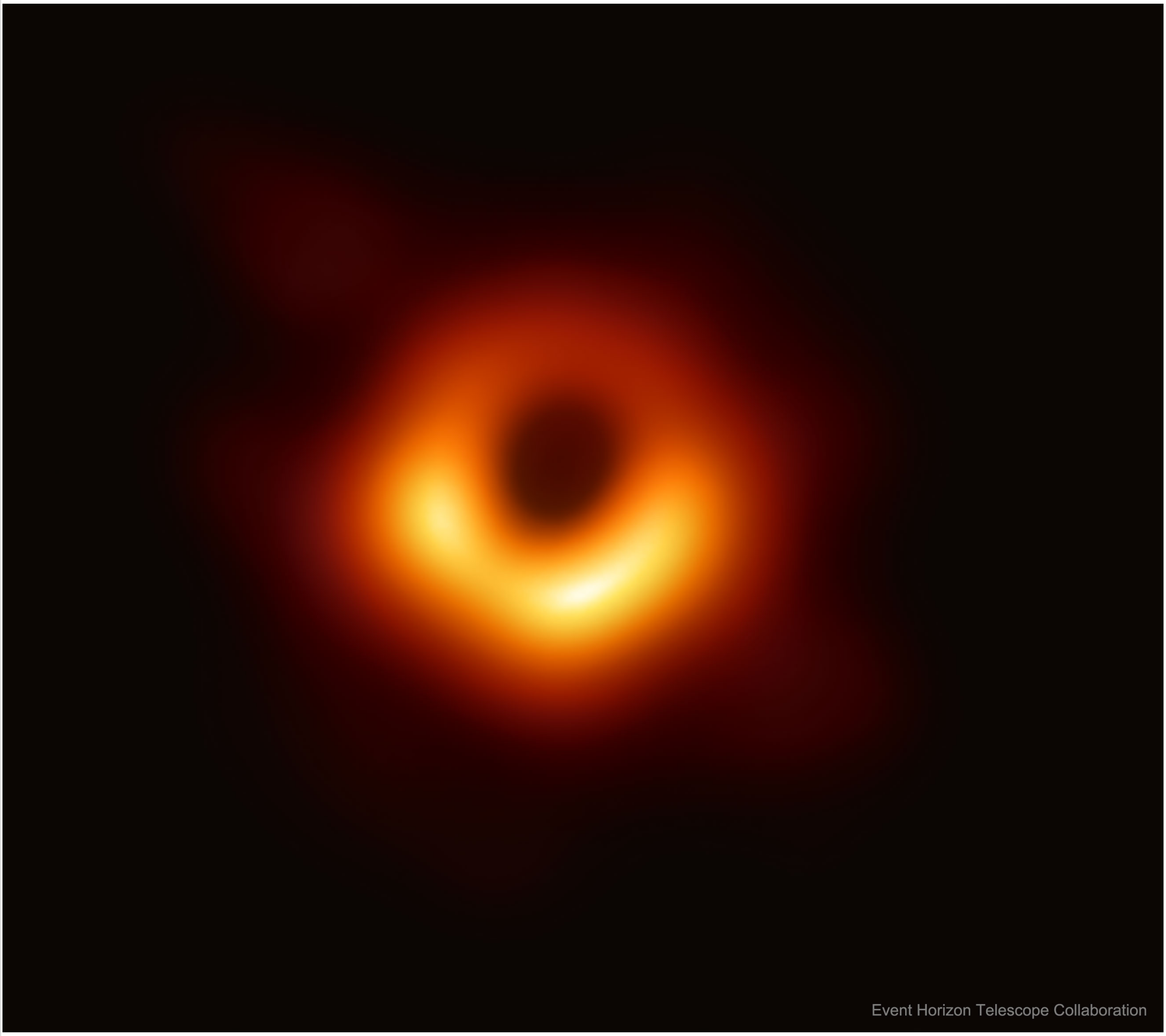
Some Calculations for the M87 Black Hole

by Robert Bee



Purely for the heck of it and because I enjoy calculating things, I had a go at using basic formulae to use the data provided by the Event Horizon Telescope project which produced the amazing image of the black hole in galaxy M87 to calculate the angular size (as seen from Earth) of the black hole (actually its event horizon) and also its mass in solar masses.
I was interested to see if my humble calculations aligned with the experts' figures.

NOTE: These are my personal calculations done to the best of my mathematical ability. If any reader can find an error in them, I would greatly appreciate your feedback so I might make a correction.

So, here is the data provided by EHT team:
EH Diameter: D = 38 x 10^9 km
Black Hole (BH) Mass: M = 6.5 x 10^9 M_0. (M_0 is 1 solar mass)

Distance from Earth to M87: d = 53.49 x 10^6 light years.
1 light year = 9.461 x 10^12 km
Gravitational Constant: G = 6.67 x 10^(-11) m^3/kg/s^2

Angular size of the Event Horizon:

Let the angular size of the EH be α:
Using the geometric formula: D(km) = α (radians) x d (km),
Then α (radians) = D/d.
Using the numbers above (in appropriate units):
α (radians) = (38 x 10^9 km)/(53.49 x 10^6 x 9.461 x 10^12 km)
= 0.075089 x 10^(-9) radians
Now 1 arcsec = pi/648,000 rad. Reversing this, we get: 1 rad = 206,264.806 arc sec.
Therefore, the apparent angular size of the Event Horizon, seen from Earth, α, is:
α = 0.075089 x 10^(-9) x 206,264.806, giving
α = 15.49 x 10^(-6) arc seconds.

That's a very small angle. To put that into perspective, if we assume that a human hair has a thickness of 0.1mm (which is average for a human hair), **how far away would you have to place a human hair for its thickness to have the same angular size as the M87 Event Horizon?**

Using the same technique as above:
Distance d x α = 0.1mm = 10^(-4) m
That is, d = 10^(-4)/α
But for the EH, α = 0.075089 x 10^(-9) radians
Substituting, we get: d = 10^(-4)m/(0.075089 x 10^(-9) radians)
= 13.32 x 10^5m
= 1,332 km.

Try to imagine that. I can't. That's more than the distance from Sydney to Adelaide.

Now to calculate the mass of the black hole.

There is a formula for the escape velocity of an object (say a rocket) from a planet.
v = sq.root (2GM/r). or v = (2GM/r)^(1/2)
where G is the gravitational constant, M is the mass of the planet, r is the distance of the object from the centre of the planet.

This same formula applies to the escape velocity from a black hole, but that velocity is set at greater than the speed of light c.
The diameter of the Event Horizon of a black hole is called the Schwarzschild Radius and its value is calculated by setting the escape velocity equal to c.
So, by rearranging the equation for the escape velocity, we get an expression for the Schwarzchild radius of:

R_s = 2GM/c^2

Now the EHT project has given us R_s (half of 38 billion km = 19 x 10^9 km = 19 x 10^12 m) and we know G and c.
So let's calculate the mass of the black hole.

M = (R_s x c^2)/2G
= [(19 x 10^12) x (3 x 10^8)^2]/[2 x 6.67 x 10^(-11)]
= 12.8186 x 10^39 kg

Now the mass of our Sun M_0 = 1.9891 x 10^30 kg.
Divide this into the value above for M and you get:

M = 6.44 x 10^9 (or 6.44 billion) times the mass of our Sun M_0.

That's very close to the reported value of the black hole's mass of 6.5 billion solar masses.

And I think that's pretty cool.